

# **Realized Volatility and Price Determinants of Bitcoin**

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## ABSTRACT

This paper presents an analysis of Bitcoin Price and Realized Volatility. Particularly, this work is concerned with price dynamics of Bitcoin crypto-currency and examines the relationships between Bitcoin Price and popularity of the Blockchain technology, as well as the relationship between Bitcoin Realized Volatility and overall market volatility measured by the CBOE Volatility Index. The mentioned relationships are considered through the lens of ARIMA and Granger Causality statistical models. We find that an ARIMA(1,1,1) model very well describes the recent price dynamics of Bitcoin and can serve as a solid ground for making future forecasts. Furthermore, we find that there is a bidirectional causality between Bitcoin Price and Blockchain popularity, and a unidirectional causality relationship between Bitcoin Realized Volatility and the CBOE Volatility Index.

**Keywords:** bitcoin price, bitcoin volatility, blockchain, VIX

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## **1. Introduction**

Bitcoin is a peer-to-peer digital currency that operates under decentralized standards and aims to provide a digital alternative to physical cash. As to the question of whether or not Bitcoin and/or other crypto-currencies can or will replace traditional fiat currency, there is an ongoing debate among monetary theorists, economists and politicians. A separate debate on whether Bitcoin and its counterparts can/should be considered as currency or as a separate group of digital assets is also relevant. This being said, it is obvious that in order to dig deeper into the roots of crypto currencies and draw practical implications, let alone construct valid models explaining their behavior, we should start by introducing some background information about Bitcoin and try to understand how exactly it operates.

Bitcoin was launched in 2009 by an individual or a group of individuals under the pseudonym “Satoshi Nakamoto”. It was launched as an electronic payment system based on a mathematical proof of work approach, which serves as the foundation of the consensus mechanism behind Bitcoin. (CNN Money, 2018) It is important to mention that the software code for Bitcoin client is open source which enabled the creation of a number of other crypto currencies. The main value proposition of Bitcoin was to provide a secure and decentralized electronic means of exchange, independent of any government or central bank. In the absence of any central authority, Bitcoin has solved the “double spending” problem by a beautiful combination of cryptographic solutions and economic

incentives. This approach upholds the legitimacy of the Bitcoin transactions and at the same time provides complete independence and disintermediation.

The technology that enables and facilitates the constant flow of transactions in Bitcoin is called Blockchain. As mentioned, one of the fundamental characteristics of Bitcoin is the absence of a central ledger, that is, transactions are not verified and recorded by a central authority, but rather this role is taken up by the nodes of the network. Blockchain creates a medium for transactions where each and every participant maintains his/her copy of the record of transactions. Through its consensus mechanism, Blockchain makes it the participants' best interest to record the correct stream of transactions and excludes the possibility of fraud.

Another underlying characteristic of Bitcoin is its limited supply. Unlike traditional fiat currencies, the supply of which is constantly monitored and controlled by central banks, Bitcoin's supply is predetermined by its underlying algorithm. The supply of Bitcoin is set to peak a little short of 21 million coins, although it is hard to predict when the last Bitcoin will be mined.

This paper is mainly interested in elaborating on factors that influence Bitcoin Price and Bitcoin Realized Volatility, as well as unveiling the relationships between Bitcoin Price and public enthusiasm for Blockchain(measured by Google Trends data) and the relationship between Bitcoin Realized Volatility and overall market volatility (measured by Cboe Volatility Index). Put in a more concrete way, this paper answers the following

research question: What are the Bitcoin Price and Volatility determinants and what dynamics do they follow?

Firstly, the understanding of the Bitcoin price movements is essential for investors or potential investors who want to gauge the determinants of price fluctuations of cryptocurrencies as well as the patterns that determine them. On the other hand, studying Bitcoin price movements is relevant for regulators and governments as well. With a better understanding of the price movements, better regulatory approaches and public policies can be designed to increase the efficiency and productivity of this new investment vehicle. Secondly, by understanding of the relationship between Bitcoin Price and Blockchain popularity, the regulators and investors can correctly link these two concepts and respond accordingly, whether making an investment decision or designing a regulatory framework. Lastly, the study of Bitcoin volatility is of utmost importance for anyone concerned with Bitcoin or other crypto-currencies. Bitcoin is known for its unpredictable price fluctuations and it is crucial to study Bitcoin volatility in order to predict and appropriately respond to those fluctuations.

## **2. Literature Review**

Since the creation of Bitcoin, the number of scholarly articles and research papers explaining its behavior has been growing steadily. After the already mentioned price rally of Bitcoin, the number of such publications has spiked. This being said, it is also important to note that most papers are concerned with the economic analysis of Bitcoin

crypto-currency, particularly, addressing whether or not it is a currency or a digital asset. In the paper “Is Bitcoin a Real Currency? An Economic Appraisal” Yermack (2013) analyzes the characteristics of Bitcoin and presents arguments as to why it cannot represent an alternative to traditional currency. Among those objections are the small transaction volumes, risks of hacking attacks and price volatility.

There is also a significant number of scholarly articles addressing the price fluctuations of Bitcoin and other crypto-currencies. In their paper “The economics of BitCoin price formation”, Ciaian, Rajcaniova and Kancs (2015) examine the determinants of Bitcoin price. The authors take into account factors that influence traditional currencies, as well as look at Bitcoin as a digital asset. Viewing Bitcoin as a currency, Ciaian et al. examined the contribution of supply and demand market forces to the formation of Bitcoin price. On the other hand, they also examined the relevance of attractiveness for investors as a possible contributor to the price determination of Bitcoin. The latter assumes that Bitcoin has characteristics of a digital asset. The conclusion is that Bitcoin price is largely determined by supply and demand forces, although attractiveness as a speculative asset is also significant.

Another paper elaborating on the price determinants of Bitcoin is “The technology and economic determinants of crypto-currency exchange rates: The case of Bitcoin”, where Li and Wang (2017) use ARDL (Autoregressive Distributed Lag) model to examine the price series of Bitcoin. The authors conclude that in the long term, Bitcoin price is more

sensitive to economic fundamentals and less sensitive to technological factors, such as advances in mining technology.

Moreover, there are quite a few papers linking Bitcoin price to Bitcoin popularity or search frequency on the internet. For instance, in the paper “BitCoin meets Google Trends and Wikipedia: Quantifying the relationship between phenomena of the Internet era”, Kristoufek (2013) analyzes the relationship between the fluctuations in the popularity of Bitcoin on the two mentioned platforms and the fluctuations in the price of Bitcoin relative to traditional currencies. The author establishes correlation and causation relationship between the crypto-currency’s popularity and its price fluctuations.

There is a relatively smaller amount of work dedicated to examining Bitcoin volatility. In her work “Bitcoin, gold and the dollar – A GARCH volatility analysis”, Dyhrberg (2016) uses GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) models to analyze Bitcoin volatility. Comparisons were made between Bitcoin and Gold, as well as Bitcoin and US Dollar, in terms of usefulness in risk management. The author concludes that Bitcoin can be somewhere in the middle of Gold and US Dollar when it comes to its characteristics as a store of value and medium of exchange.

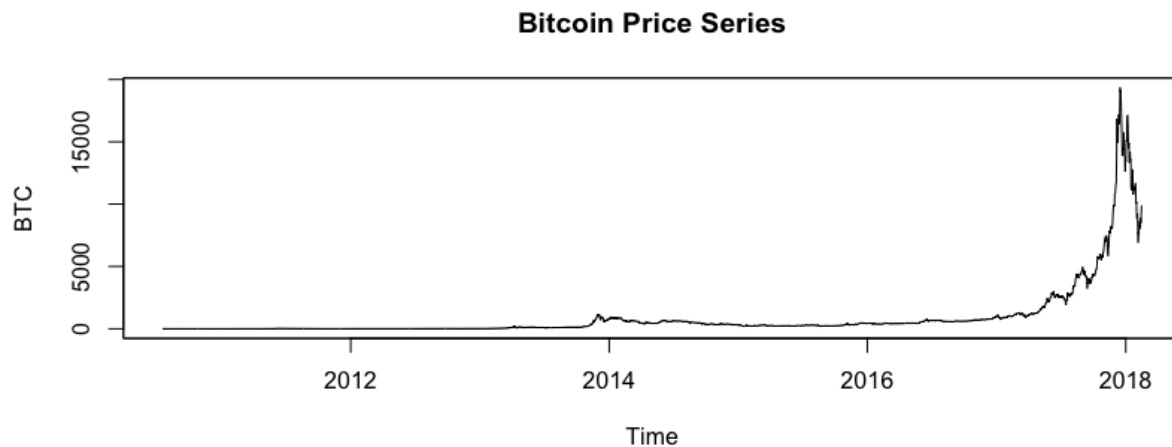
This paper contributes to the discussion by using different models to explain the mentioned relationships as well as present new modifications of the discussed relationships. Namely, an ARIMA model explaining Bitcoin price fluctuations is used, as well as Granger Causality models for explaining causation relationships between relevant time series.



### 3. Data and Descriptive Statistics

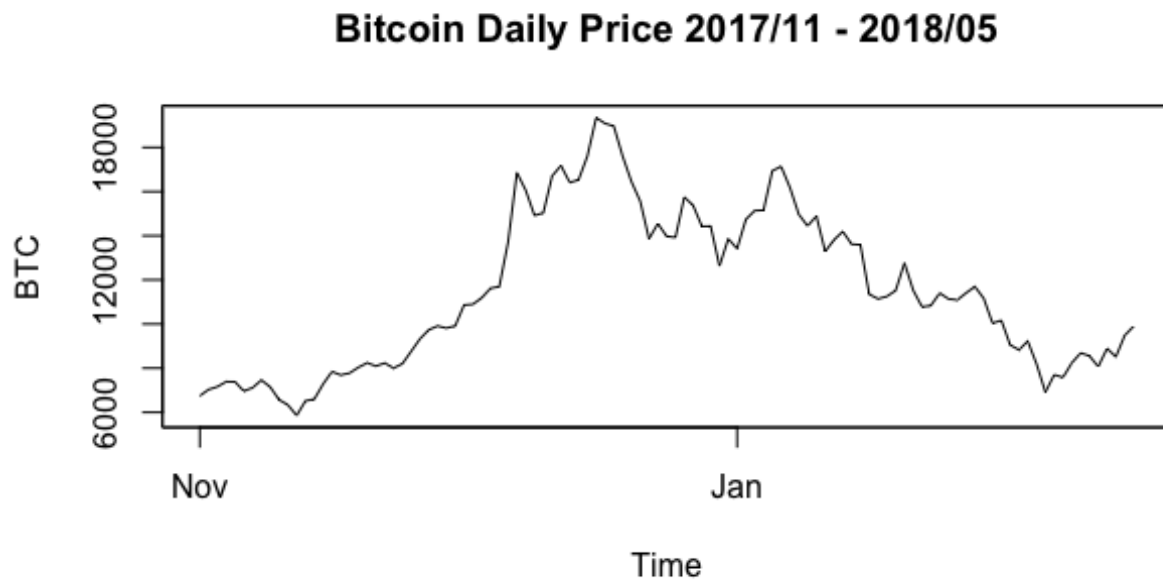
In the framework of this paper, we use the historical series for Bitcoin/USD exchange price, Bitcoin Volatility Index, Cboe Volatility Index and the Google Trends data on Blockchain popularity.

For analyzing Bitcoin price, we use daily Bitcoin/USD exchange rate data starting from the inception of the Bitcoin crypto currency in 2010.



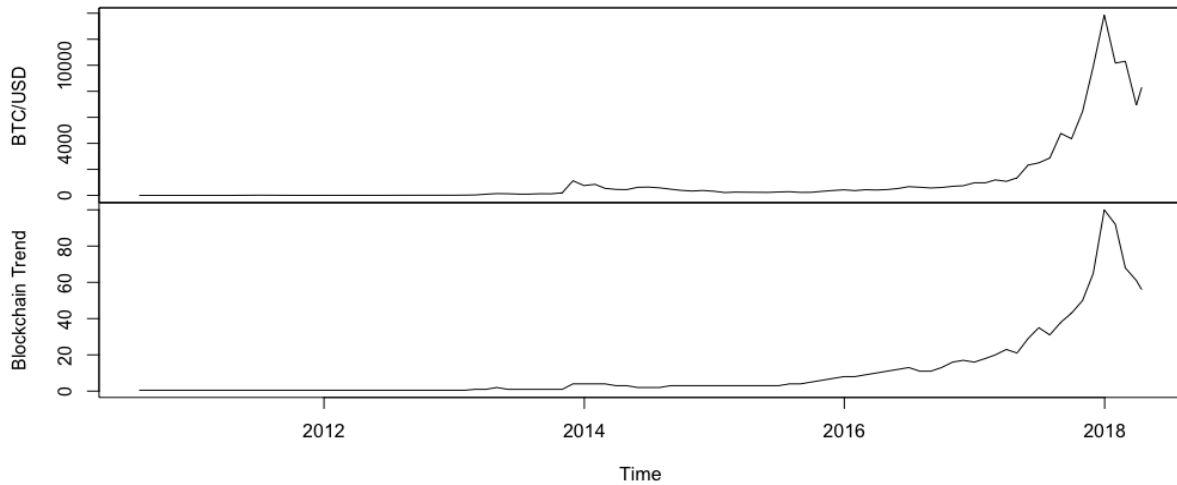
Data is taken from “Coindesk” and represents the average of Bitcoin/USD exchange rates across all major crypto-currency exchanges. The figure above captures the historical price movements of Bitcoin since it was first launched in 2010. The unexpected price rally that started in 2017 was responsible for the major public interest in Bitcoin. On December 18<sup>th</sup>, 2017 Bitcoin price hit an all time high just short of \$20,000. The price hike reversed at \$19,498.63 per Bitcoin. In terms of analyzing Bitcoin price and fitting ARIMA models, we also use a subset of the daily price series from 01/11/2017 to 15/04/2018, as

the price fluctuations of Bitcoin present different dynamics in different periods of time. The mentioned time interval is the most recent and the most fit for constructing ARIMA models. The graph of the mentioned subset is presented below.



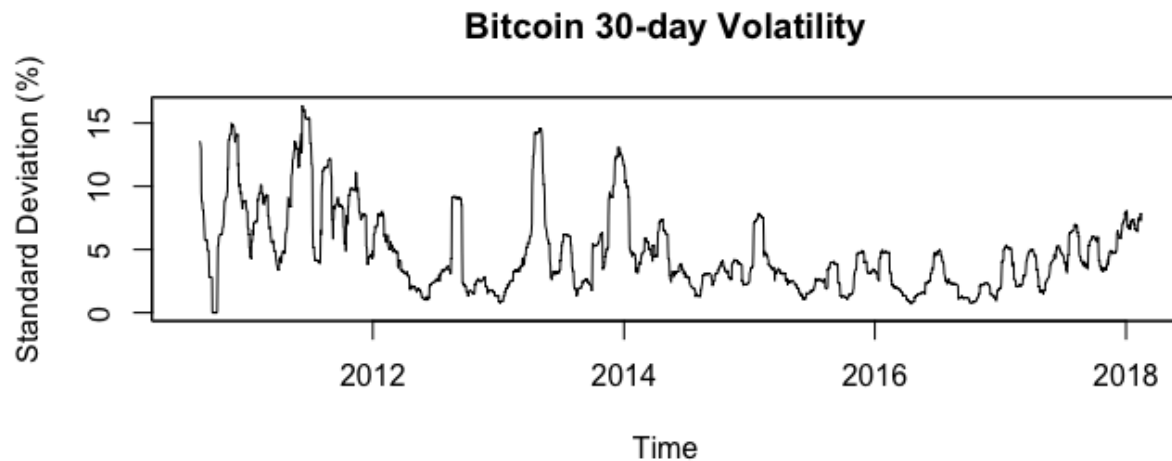
In terms of analyzing Bitcoin price and its determinants, in the framework of this paper, another relevant series is the Blockchain popularity measured by Google Trends data. This data measures the frequency of the searches of the word “Blockchain”, as well as other related concepts.

**Bitcoin Price and Blockchain Popularity**



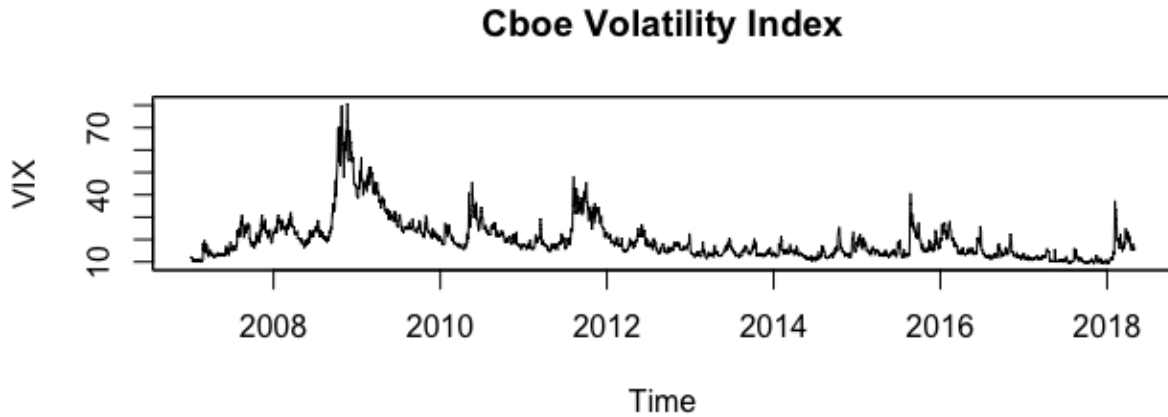
The figure above shows the Blockchain popularity and Bitcoin price series together. It is obvious from the plot that there is a high correlation between the two series and it is reasonable to assume that there is a causation relationship between Bitcoin Trend and BTC/USD exchange price. Formal analysis will be conducted to unveil this relationship.

In order to analyze possible determinants of Bitcoin volatility, we use Bitcoin price 30-day annualized volatility data. For obtaining this series, we calculate the 1-month rolling standard deviation of the returns. The returns, in their turn, are calculated as the natural logarithm of the ratio of successive price points. The graph of the series is presented below:



The graph of the series points out that Bitcoin volatility has stabilized over the past two years and is now at significantly lower levels than before.

The Cboe Volatility index (VIX) was chosen as a representative measure of the overall market volatility. VIX is calculated based on a wide range of S&P 500 index option prices and represents the market's expectation of 30-day volatility. VIX is also referred to as the "investor fear gauge". VIX uses a confidence interval of one standard deviation of a normal probability curve (i.e. 68%). The graph below summarizes the historical price series of the VIX.



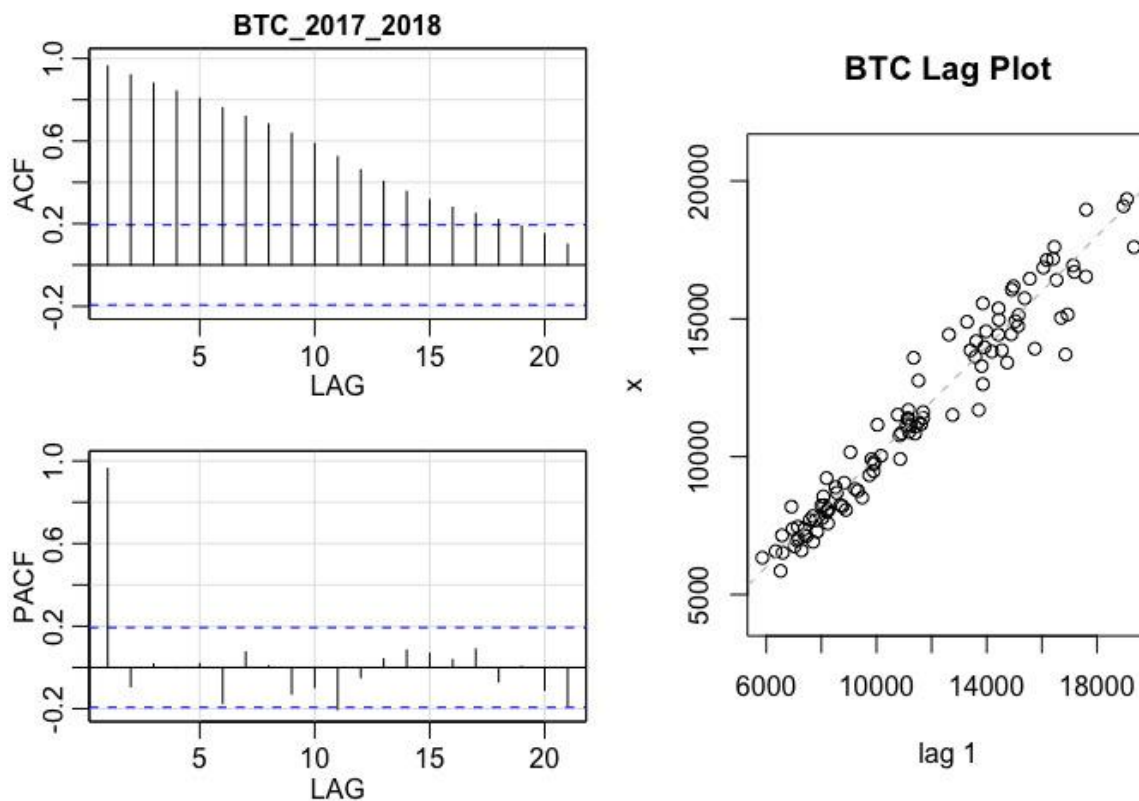
In the context of this analysis, the Cboe VIX index will serve as a benchmark of overall market volatility.

#### **4. Models and Methodology**

Regarding the methodology and construction of specific models explaining the relationships under consideration, there are several approaches that are taken up in this study. For obtaining a meaningful model explaining price behavior of Bitcoin, we fit an ARIMA model to a recent subset of Bitcoin price data to explain the price movements with an Autoregressive Integrated Moving Average model. The ARIMA model is a general version of the ARMA model. Both are fitted to time series data to unveil the dynamics of the data and make forecasts. The AR part of the model is responsible for the regression of the data on its own lagged values. The MA part of the model indicates that the errors of the values are linear combinations of past errors. Lastly, the Integration

refers to the degree of differencing of the data after which it becomes stationary. It is important to note that for a meaningful ARIMA model, a subset of Bitcoin price series ranging from November,2017 to May, 2018 is taken, as this portion of the data demonstrates patterns best suited for ARIMA modelling.

Before deciding whether we need an ARIMA or ARMA model, it is important to determine whether or not the series in question is stationary.



As we can observe from the lag plot and the ACF and PACF functions of the price data, the series is clearly not stationary because of the slow decay in ACF and apparent patterns in the lag plot. This being said, we also run an Augmented Dickey-Fuller Test

that tests the null hypothesis of data being not stationary. The results of the test are presented below:

### Augmented Dickey-Fuller Test

```
data: BTC_2017_2018
Dickey-Fuller = -1.1887, Lag order = 4, p-value = 0.905
alternative hypothesis: stationary
```

As we can see from the formal test, the data is not stationary as the p-value is larger than 0.05 and the null hypothesis is not rejected. Hence, differencing is necessary to secure stationarity, which is why we use ARIMA rather than ARMA model.

To determine the parameters of ARIMA(p,d,q) model, we use two methods. First, an automated algorithm for fitting an ARIMA model to data is used. This process is specifically designed to yield parameters of the model that minimize Akaike Information Criteria(AIC) and Bayesian Information Criteria(BIC). The second method for determining the best model for the Bitcoin price series is overfitting. The idea of overfitting is to continuously add parameters to the model until the next parameter is no longer significant. In this case, the two methods coalesce in their conclusions and yield an ARIMA(1,1,1) model for the Bitcoin price series. The results and interpretations of the model are presented in the next section.

With regards to determining the causality relationship between Bitcoin price and Blockchain Trend, as well as Bitcoin Volatility and VIX, in the framework of this

analysis, Granger Causality models are constructed. Granger Causality is a statistical concept of causality that is used in prediction. It is important to note that Granger Causality is different from regular “cause and effect” relation and shows a certain type of correlation, rather than true causation. The formal interpretation of Granger Causality between variables X and Y is that in case of X Granger-causing Y, the past values of X contain information that is useful in predicting the future values of Y. Statistically, Granger Causality represents a combination of t-tests and F-tests on lagged entries of variables. The usage of Granger Causality models to explain the relationships between the mentioned series, as well as other variables, can be seen in UC Berkeley Undergraduate Thesis paper “Analyzing Bitcoin Price Volatility” by Julio Cesar Soldevilla Estrada(2017).

It is important to mention that Granger causality test only yields meaningful results when stationary data is taken as an input. In this regard, variables need to be tested for stationarity, and in case of non-stationary data, necessary transformations are needed to achieve stationarity. The outputs of formal Dickey-Fuller tests of monthly Bitcoin price series and Blockchain trend data are presented below.

### **Augmented Dickey-Fuller Test**

```
data: monthly_btc
Dickey-Fuller = 2.3787, Lag order = 4, p-value = 0.99
alternative hypothesis: stationary
```



## Augmented Dickey-Fuller Test

```
data: blockchain$Blockchain  
Dickey-Fuller = 4.6889, Lag order = 4, p-value = 0.99  
alternative hypothesis: stationary
```

As the formal analysis shows, both of the series are not stationary, which is why differencing is necessary before running Granger Causality tests. The ACF and PACF functions along with lag plots of the Bitcoin monthly series and Blockchain series are presented in the appendix. Both of the series show stationarity after differencing the log of the series.

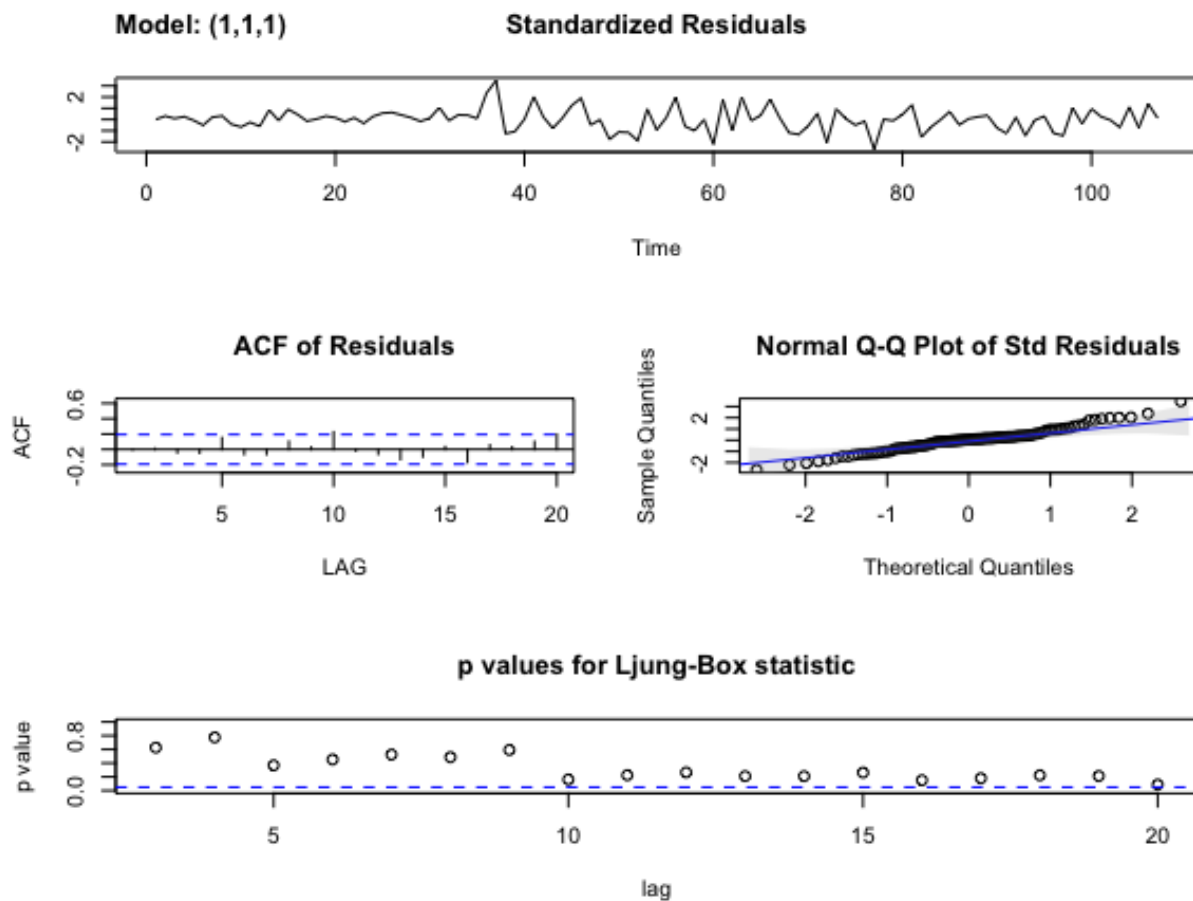
Analogous analysis is carried out for the Bitcoin volatility series and VIX series. Both of these series exhibit stationarity. For the results of the formal Augmented Dickey-Fuller tests please refer to the appendix. The results and interpretations of the Granger causality models are presented in the next section.

## 5. Results

In this section, the results and interpretations of the models mentioned above are presented. First, we elaborate on the results of the ARIMA (1,1,1) model fitted to a subset of Bitcoin daily price series. Next, the results and interpretations of the Granger Causality models between Bitcoin price and Blockchain trend and Bitcoin volatility and VIX index are presented.

## ARIMA (1,1,1)

As mentioned in the previous section, an ARIMA (1,1,1) was determined to be the best fit for the Bitcoin price data under consideration. The model was chosen with the minimization of AIC and BIC criteria, as well as using the method of overfitting. Below is presented the residual analysis for determining whether the residuals are Gaussian white noise and if we have found the best model.



The residual analysis shows that the model satisfies all the conditions. There is no pattern in the Standardized Residuals plot, the ACF of the residuals shows no autocorrelation, Q-

Q plot confirms the normality of the distribution of residuals and the p-values of the Ljung Box statistic show that there is no correlation in the residuals and they follow a White Noise(WN) process.

Given the sufficiency of the model, we can move on to the results and the implications of it. The table below summarizes the ARIMA (1,1,1) under consideration.

	Estimate	SE	t.value	p.value
ar1	-0.6665	0.1980	-3.3664	0.0011
ma1	0.8001	0.1571	5.0922	0.0000
constant	28.6606	86.9412	0.3297	0.7423

As we can observe from the coefficient table, our model yields an AR(1) parameter coefficient of -0.6665 and an MA(1) parameter coefficient of 0.8001. Given these findings we can state that the price of Bitcoin within this model can be approximated with the following formula:

$$Y_t = -0.6665Y_{t-1} + e_t + 0.8001e_{t-1}$$

The AR(1) coefficient means that the value of Bitcoin price depends on its own value one lag apart with a coefficient of -0.6665. The amplitude of the ar1 coefficient also determines its rate of convergence to the mean. Thus, the absolute value of the coefficient equaling roughly 0.7 means that Bitcoin price fluctuations do not quickly converge to the mean. This can be shown by referring at simulated values of AR(1) models. The significance of the ma1 parameter shows that the value of Bitcoin price also depends linearly on past values of the stochastic term.

## *Granger Causality*

In terms of unveiling the causation relationship between Bitcoin price and Blockchain trend, we construct a Granger Causality model. It is important to mention that the Bitcoin daily data is aggregated into a monthly series of closing prices in order to suite the analysis and match the frequency of the Blockchain trend data. As already mentioned, we need stationary data as an input for Granger tests. After taking the difference of the log series for both of the variables, they exhibit stationarity. The formal Augmented Dickey-Fuller test results for both series can be found in the appendix.

Using the log and differencing transformations we obtained stationary series upon which the Granger Causality models are constructed. The following table summarizes the results of running Granger Cuasality to determine whether Bitcoin price Granger-causes Blockchain popularity:

`Granger causality H0: BTC do not Granger-cause Blockchain`

```
data: VAR object var1
F-Test = 2.4181, df1 = 9, df2 = 126, p-value = 0.01444
```

The null hypothesis of the test is that BTC does not Granger-cause Blockchain. As the p-value = 0.01444 < 0.05, we can reject the null hypothesis with 95% confidence interval and conclude that Bitcoin price does Granger-cause Blockchain popularity. Next, we run the reverse test to see whether Blockchain similarly Granger-causes BTC. The table below summarizes the results of the test:

Granger causality H0: Blockchain do not Granger-cause BTC

```
data: VAR object var1  
F-Test = 1.9752, df1 = 9, df2 = 126, p-value = 0.04743
```

The null hypothesis here is that Blockchain series does not Granger-cause BTC. As the p-value = 0.04743 < 0.05, we can reject the null hypothesis with 95% confidence interval and conclude that Blockchain does Granger-cause BTC. To sum up the results of the two tests presented above, it is noteworthy that Granger Causality model yields bidirectional Granger causality between Bitcoin monthly price and Blockchain trend measured by Google Trends data. The main inference from this finding is that past values of Bitcoin price are valuable predictors of future values of Blockchain popularity and past values of Blockchain trend are good predictors of future values of Bitcoin price.

When it comes to uncovering the relationship between Bitcoin volatility and overall market volatility measured by VIX, we follow the exact same steps as in constructing the previous Granger causality model. Here, both of the series are stationary and there is no need to make transformations of the data. First, we run a test to determine whether Bitcoin volatility Granger-causes VIX. The following table summarizes the results.

Granger causality H0: btc\_vol do not Granger-cause vix

```
data: VAR object var2  
F-Test = 2.5145, df1 = 19, df2 = 5364, p-value = 0.0002891
```

The null hypothesis of this test is that `btc_vol` does not Granger-cause `vix`. As the p-value = 0.0002891 < 0.05, we reject the null hypothesis with 95% confidence interval and conclude that Bitcoin volatility does Granger-cause overall market volatility measured by VIX index. Again, we run the reverse test to determine if the opposite Granger-causality exists. The next table summarizes the findings of the test.

```
Granger causality H0: vix do not Granger-cause btc_vol  
data: VAR object var2  
F-Test = 1.1103, df1 = 19, df2 = 5364, p-value = 0.332
```

The null hypothesis here is that `vix` does not Granger cause `btc_vol`. P-value = 0.332 > 0.05 and we fail to reject the null hypothesis. We conclude that VIX does not Granger-cause Bitcoin volatility. All in all, the two Granger causality tests yield unidirectional causality towards VIX index. This means that past values of Bitcoin volatility are meaningful predictors of the overall market volatility measured by VIX index, while past values of VIX are not useful in predicting future values of Bitcoin volatility.

## 6. Conclusions

The statistical models and analysis carried out in the framework of this study yield several interesting conclusions. Firstly, we found that the recent price dynamics of Bitcoin can be very well explained within ARIMA(1,1,1) model, which can provide

meaningful insight about possible future dynamics of Bitcoin price. This result can be useful for forecasting Bitcoin price into the short and medium term future. The convergence of Bitcoin price towards an ARIMA process can also have indirect connection with its reduced volatility during the past year. A suggested explanation for this connection is that the price of Bitcoin becomes possible to estimate using relatively simple statistical models and people become more steadfast towards expected values of Bitcoin price.

Secondly, this paper shows Granger causality relationship between Bitcoin price and Blockchain popularity. The formal tests yield bidirectional Granger causality between these two series which means that past values of both of these series contain valuable information to predict forecast future values of the other series. This finding is consistent with the conclusions of similar studies. One important implication of this fact is that despite the constant change in Bitcoin price dynamics, its correlation and causation relationship with the popularity of its underlying technology is preserved.

Thirdly and lastly, this paper elaborates on the Granger causality relationship between Bitcoin realized volatility and CBOE VIX index. The statistical tests point out that there is a unidirectional Granger causality relationship. That is, past values of Bitcoin volatility contain information for forecasting future values of the VIX, while the reverse is not true. A possible implication of this finding is that overall market volatility does not affect Bitcoin volatility. This would mean that shocks in the VIX would not result in similar shocks in Bitcoin volatility. As a suggested hypothesis, it would be sensible to

consequently view Bitcoin as a hedging instrument or even a “safe haven” asset like gold. Moreover, this characteristic would make Bitcoin a decent candidate for serving as a diversification tool in an investment portfolio. Of course, this is only a consideration and it may be an interesting topic for further research.

As mentioned in the Introduction section, this paper and its findings can be useful primarily for investors and regulators, as well as anyone who is interested in Bitcoin and/or other crypto-currencies. Taking into account the relatively recent popularity of crypto-currencies, the understanding of price and volatility determinants and their behavior is of utmost relevance for anyone concerned with these instruments. Again, as mentioned, the illustrated work shows some interesting findings and sets a small stage for further research.

I agree that my paper be posted on the library database for an open access to the AUA Community.



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## 8. Appendix

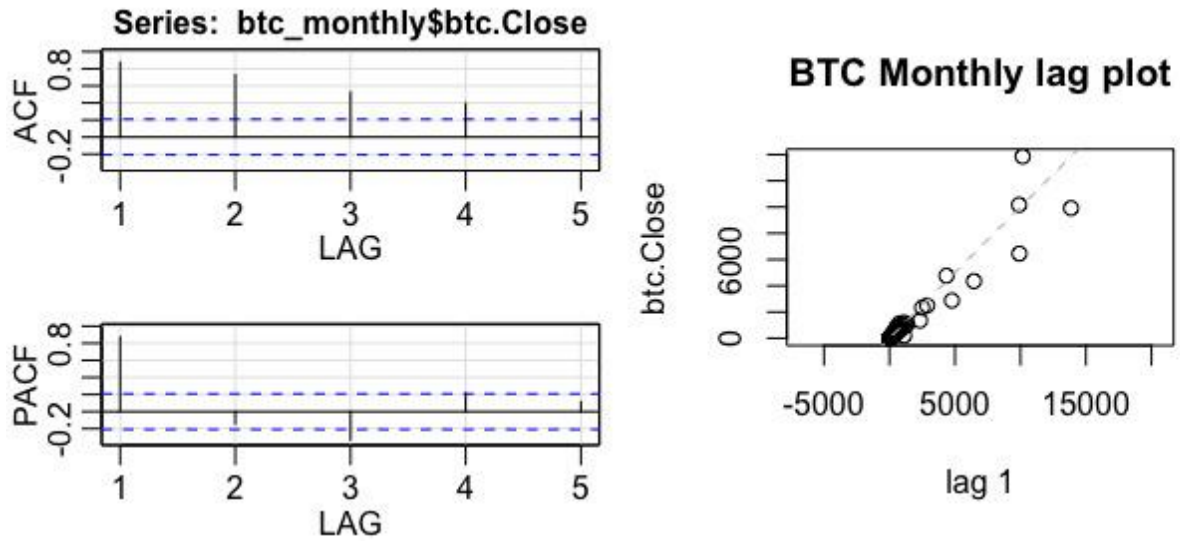


Figure 1: ACF and PACF and Lag plot for Bitcoin monthly data.

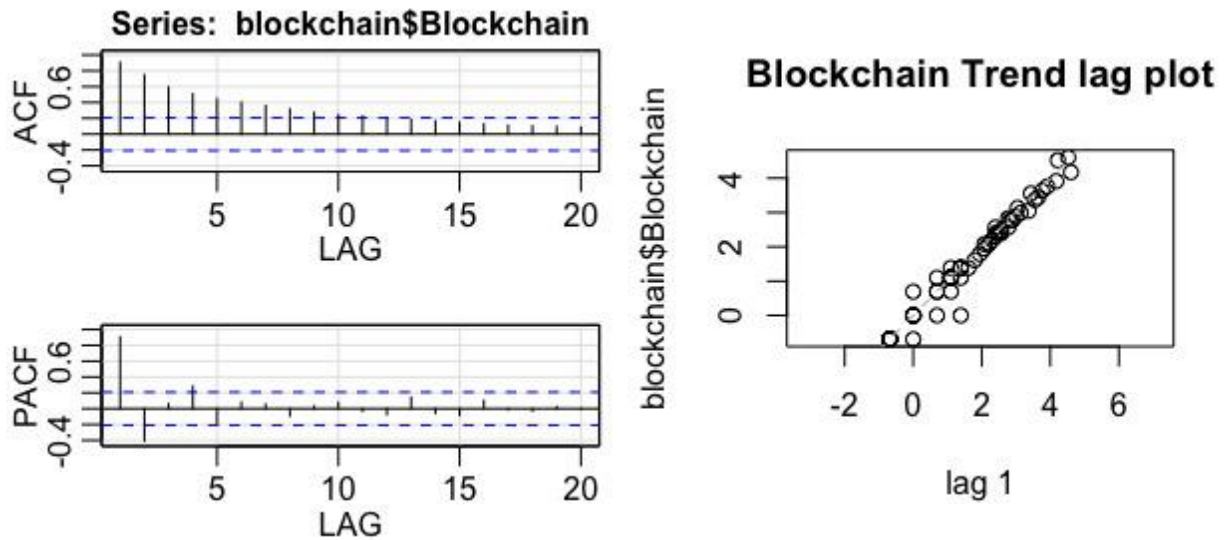


Figure 2: ACF and PACF and lag plot of Blockchain trend data.

### Augmented Dickey-Fuller Test

```
data: diff(btc_monthly$btc.Close)
Dickey-Fuller = -4.9107, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary
```

Figure3: Augmented Dickey-Fuller Test for monthly Bitcoin price transformed by taking the natural logarithm and then differencing. The test shows that the data is stationary as the p-value  $< 0.01 < 0.05$  and the null hypothesis of unit root is rejected.

### Augmented Dickey-Fuller Test

```
data: diff(blockchain$Blockchain)
Dickey-Fuller = -5.2093, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary
```

Figure 4: Augmented Dickey-Fuller Test for Blockchain Trend data after taking natural logarithm and differencing. The test shows that the data is stationary as the p-value  $< 0.01 < 0.05$  and the null hypothesis of unit root is rejected.

### Augmented Dickey-Fuller Test

```
data: granger1$BTC
Dickey-Fuller = -6.0501, Lag order = 13, p-value = 0.01
alternative hypothesis: stationary
```

Figure 5: Augmented Dickey-Fuller Test for Bitcoin 30-day Realized Volatility. The test shows that the data is stationary as the p-value  $< 0.01 < 0.05$  and the null hypothesis of unit root is rejected.

## Augmented Dickey-Fuller Test

```
data: granger1$VIX.Close  
Dickey-Fuller = -4.4611, Lag order = 13, p-value = 0.01  
alternative hypothesis: stationary
```

Figure 6: Augmented Dickey-Fuller Test for CBOE Volatility Index (VIX) data. The test shows that the data is stationary as the p-value  $< 0.01 < 0.05$  and the null hypothesis of unit root is rejected.